Sinusoidal signals & phasors.

Modern power system operates on 3-phase alternating currents.

We will define this later in this chapter.

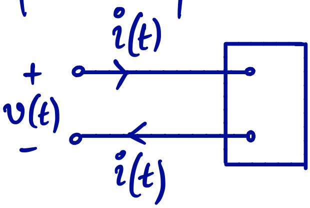
Voltage everywhere within the entire power system is a sinusoidal function of time at a particular frequency.

- In the US, that frequency is 60 Hz.
- In Europe, that frequency is 50 Hz.

Q. Can you interconnect two power systems operating at different frequencies?

A. Not directly... can be done through DC interties. We will not study that.

Consider a single "component" connected to a power system as shown :



$$v(t) = V_0 \cdot \cos(\omega t + \theta_v).$$

called the angular frequency.

$$\omega = 2\pi \cdot (\text{frequency}) \cdot \text{rad}$$

$$= 2\pi \cdot 60 \ \text{rad/s}$$

for a 60 Hz power system.

This number often comes in handy in calculations

$$\approx 377 \ \text{rad/s}.$$

Associate with the signal $v(t)$, the complex number $\overline{V} := \frac{V_0}{\sqrt{2}} e^{j\theta_v}$, known as the voltage phasor. Here, $j := \sqrt{-1}$.

Q. What is the unit of the voltage phasor?

A. Volts.

- If this component is comprised of inductive, capacitive, and resistive elements, then the current $i(t)$ can be shown to be sinusoidal with the <u>same</u> frequency as of $v(t)$.

Let $i(t) := i_0 \cos(\omega t + \theta_i)$.

Associate with $i(t)$, the current phasor

$$\bar{I} := \frac{i_0}{\sqrt{2}} e^{j\theta_i}$$

- Let's calculate the instantaneous power $p(t)$ flowing into the component.

$$p(t) = v(t) \cdot i(t)$$

$$= v_0 \cos(\omega t + \theta_v) \cdot i_0 \cos(\omega t + \theta_i)$$

$$= \frac{1}{2} v_0 i_0 \Big[\underbrace{\cos(2\omega t + \theta_v + \theta_i)}_{\text{sinusoidal with angular frequency } 2\omega.} + \underbrace{\cos(\theta_v - \theta_i)}_{\text{remains constant.}} \Big].$$

- Average power is given by

$$\langle p(t) \rangle = \frac{1}{T}\int_0^T p(t)\,dt,$$

where T is a "cycle" given by

$$T = \frac{1}{\text{frequency}} = \frac{2\pi}{\omega}.$$

Recall $p(t)$ has two components: sinusoidal with ang. freq. 2ω; time-invariant component.

Avg. of sinusoidal component $= 0$ ... convince yourself!

$$\Rightarrow \langle p(t) \rangle = \frac{1}{T}\int_0^T v_0 i_0 \cos(\theta_v - \theta_i)\,dt$$

$$= \frac{1}{2} v_0 i_0 \cos(\theta_v - \theta_i).$$

Now, let's express $\langle p(t) \rangle$ in terms of the phasors $\bar{V}$, $\bar{I}$.

Notice that $\mathrm{Re}\{\bar{V}\bar{I}^*\}$ $*$ denotes complex conjugate.

$$= \mathrm{Re}\left\{ \frac{v_o}{\sqrt{2}} e^{j\theta_v} \cdot \frac{i_o}{\sqrt{2}} e^{-j\theta_i} \right\}$$

$$= \frac{1}{2} v_o i_o \, \mathrm{Re}\left\{ e^{j(\theta_v - \theta_i)} \right\}$$

$$= \frac{1}{2} v_o i_o \cos(\theta_v - \theta_i).$$

$$= \langle p(t) \rangle$$

In words, the average power equals the real part of a complex number $\bar{S} = \bar{V}\bar{I}^*$.

We call $\bar{S}$, "complex power" or "apparent power."

Q. Is $\bar{S}$ a phasor?

A. No! $\bar{S}$ is not associated with a purely sinusoidal signal that is physically meaningful.

Write $\bar{S} = P + jQ$. Then, we call

P as real power

Q as reactive power.

P has the physical meaning that it equals the average power transferred. Q does not quite enjoy such an interpretation.

- A digression: The "root-mean-square" value of the voltage signal $v(t)$ is defined as

cycle length

$$V_{RMS} := \left(\frac{1}{T} \int_0^T v^2(t)\, dt \right)^{1/2}.$$

Let's simplify!

$$V_{RMS} = \left(\frac{1}{T} \int_0^T V_0^2 \cos^2(\omega t + \theta_v)\, dt \right)^{1/2}$$

$$= \left(\frac{V_0^2}{T} \int_0^T \frac{1}{2}\left(1 + \cos\left(2(\omega t + \theta_v)\right)\right) dt \right)^{1/2}$$

$$\boxed{\begin{aligned}&\cos 2\alpha \\ &= 2\cos^2\alpha - 1.\end{aligned}}$$

$$V_{RMS} = \left(\frac{V_0^2}{T} \int_0^T \frac{1}{2}\left(1 + \overbrace{\cos\left(2(\omega t + \theta_v)\right)}^{\text{sinusoidal with ang. freq. } 2\omega.}\right) dt \right)^{1/2}$$

$$= \left(\frac{V_0^2}{T} \int_0^T \frac{1}{2}\, dt \right)^{1/2}$$

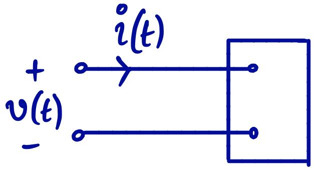
$$= \frac{V_0}{\sqrt{2}}.$$

Recall that $\bar{V} = \frac{V_0}{\sqrt{2}} e^{j\theta_v}$, which also equals $V_{RMS}\, e^{j\theta_v}$.

- Back to complex power $\bar{S} = P + jQ$.

What are their units?

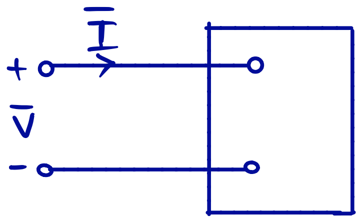
- S : volt-ampere (VA).
- P : watt (W).
- Q : volt-ampere-reactive (VAR).



transfering average power $\langle p(t)\rangle = \langle v(t).i(t)\rangle$

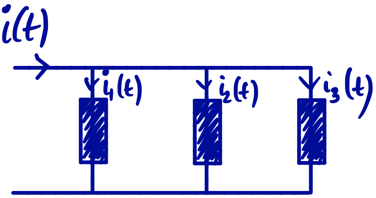
This is the real circuit with sinusoidal time domain signals.

This is a mathematical abstraction of a circuit with phasors and other complex quantities



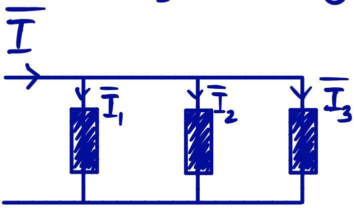
transferring complex power $\bar{S} = \bar{V}\bar{I}^*$.

The signals $v(t)$, $i(t)$ obey Kirchhoff's laws, and $p(t)$ remains conserved. For example,



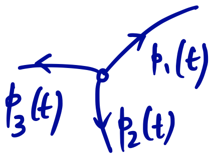
Then, $i(t) = i_1(t) + i_2(t) + i_3(t)$

The mathematical abstraction with phasors also satisfy analogous rules.



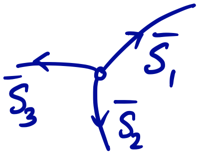
Then, $\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$.

Similarly,



$$\Rightarrow \quad p_1(t) + p_2(t) + p_3(t) = 0$$

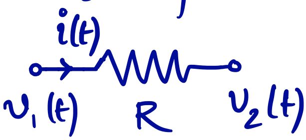
Equivalently,



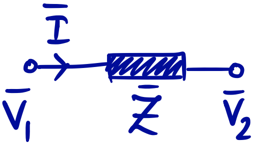
$$\Rightarrow \quad \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 0$$

The mathematical abstraction reduces manipulation with sinusoids to arithmetic on complex numbers. It is a useful formalism.

Analog of Ohm's law:



$$\Rightarrow \quad v_1(t) - v_2(t) = R \cdot i(t)$$



$$\Rightarrow \quad \bar{V}_1 - \bar{V}_2 = \bar{Z} \cdot \bar{I} \, ,$$

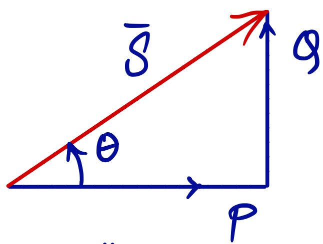
$\bar{Z}$ is called the impedance.

$\bar{Z} = R$ if it is a purely resistive element.

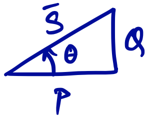
$\bar{Z} = j\omega L$, if it is a purely inductive element.

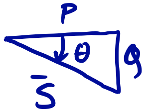
$\bar{Z} = 1/(j\omega C)$, if it is a purely capacitive element.

- Power triangle: Draw $P, Q, \bar{S}$ on the complex plane.



$\cos\theta$ is called "power factor."

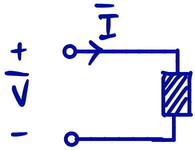
- 

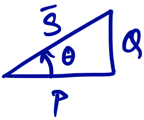
and 

both have the same power factor because $\cos\theta = \cos(-\theta)$.

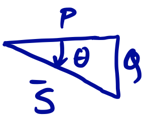
Consider a component that consumes (complex) power $\bar{S}$.

Call such a component a "load". To distinguish between the above power triangles with the same power factors, we call them by two different names.



- 

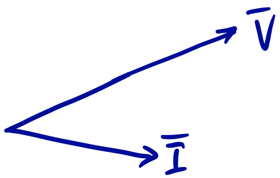
: Lagging $(Q > 0)$

- 

: Leading $(Q < 0)$.

Q. Why do we label them as "lagging" or "leading"?

To answer the question, let's plot $\bar{V}$ and $\bar{I}$ on the complex plane. Two possible cases are



$\theta_v - \theta_i > 0.$

In terms of phases, the current "<u>lags</u>" the voltage.

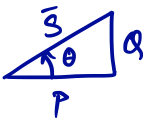
$$\bar{S} = \bar{V}\bar{I}^*$$

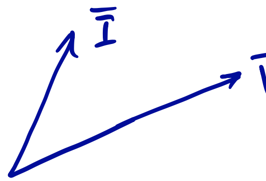
$$\Rightarrow Q = \mathrm{Im}\{\bar{V}\bar{I}^*\}$$

$$= |\bar{V}| \cdot |\bar{I}| \cdot \sin(\theta_v - \theta_i)$$

$$> 0.$$

$\Rightarrow$





$\theta_v - \theta_i < 0.$

In terms of phases, the current "<u>leads</u>" the voltage.

$$\bar{S} = \bar{V}\bar{I}^*$$

$$\Rightarrow Q = \mathrm{Im}\{\bar{V}\bar{I}^*\}$$

$$= |\bar{V}| \cdot |\bar{I}| \cdot \sin(\theta_v - \theta_i)$$

$$< 0$$

$\Rightarrow$